Solvay Brussels School
Economics & Management

ULB

Corporate Valuation and Financing
The WACC Battle!

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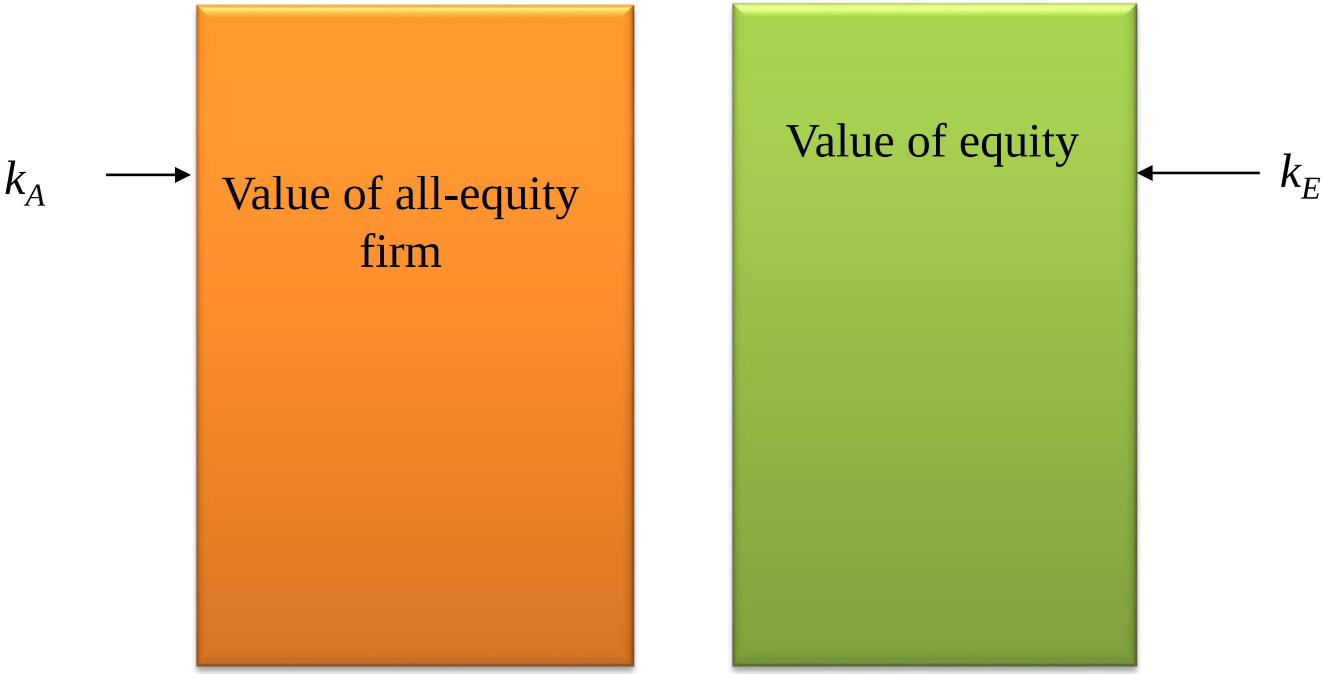
A LONG STORY MADE SHORT…

We are in this valuation context…

$$V = \sum_{t=0}^{\infty} \frac{FCF_t}{(1+k)^t}$$

With no debt…



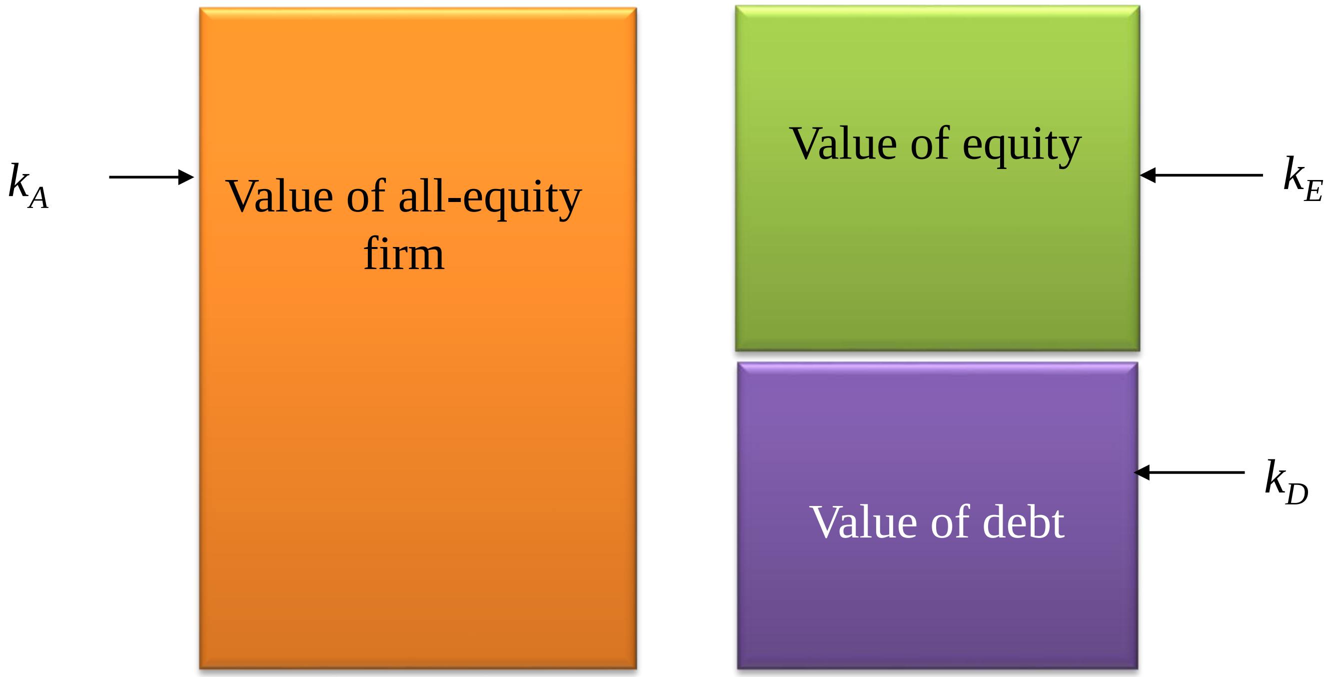
k_A → Value of all-equity firm

Value of equity ← k_E

$$k_a = k_e$$

$$V_U = \sum_{t=0}^{\infty} \frac{FCF_t}{(1 + k = k_a = k_e = WACC)^t}$$

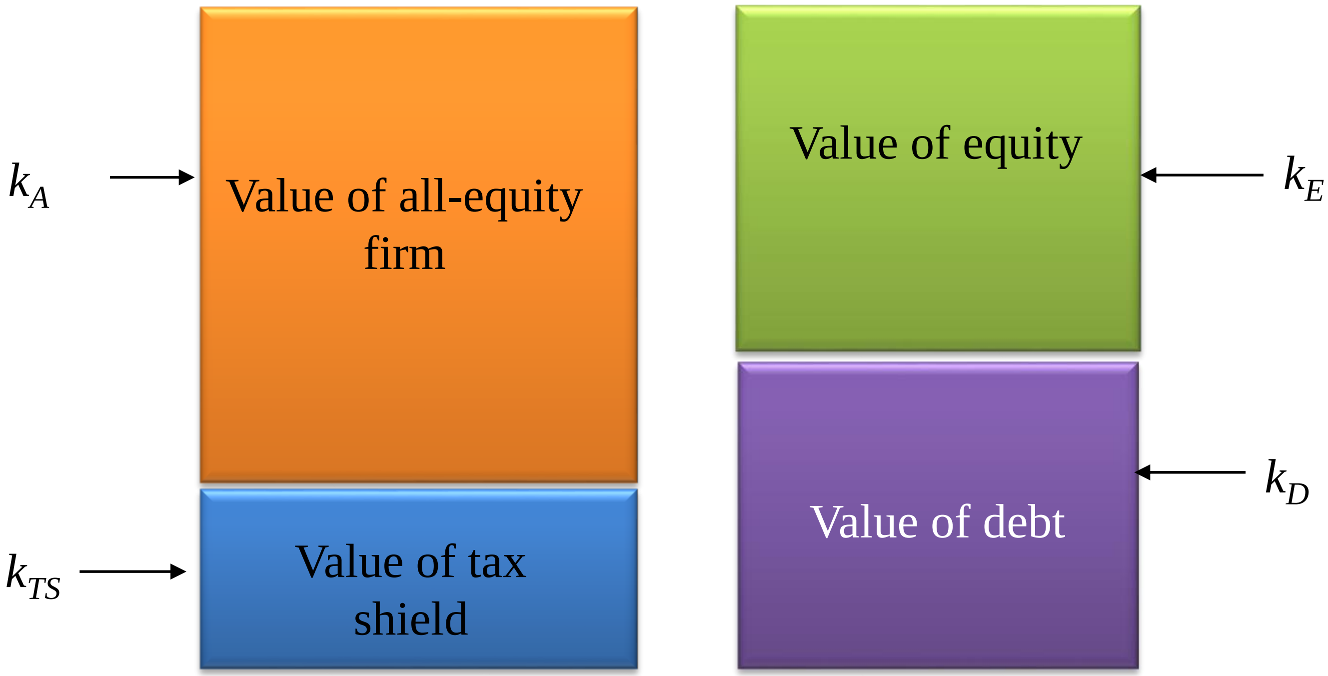
With debt, without taxes



$$k_a = k_E \frac{E}{V} + k_D \frac{D}{V}$$

$$V_L = V_U \equiv V$$

With debt and taxes



k_A → Value of all-equity firm

k_{TS} → Value of tax shield

Value of equity ← k_E

Value of debt ← k_D

$$k_a \frac{V_U}{V_L} + k_{TS} \frac{V_{TS}}{V_L} = k_E \frac{E}{V_L} + k_D \frac{D}{V_L}$$

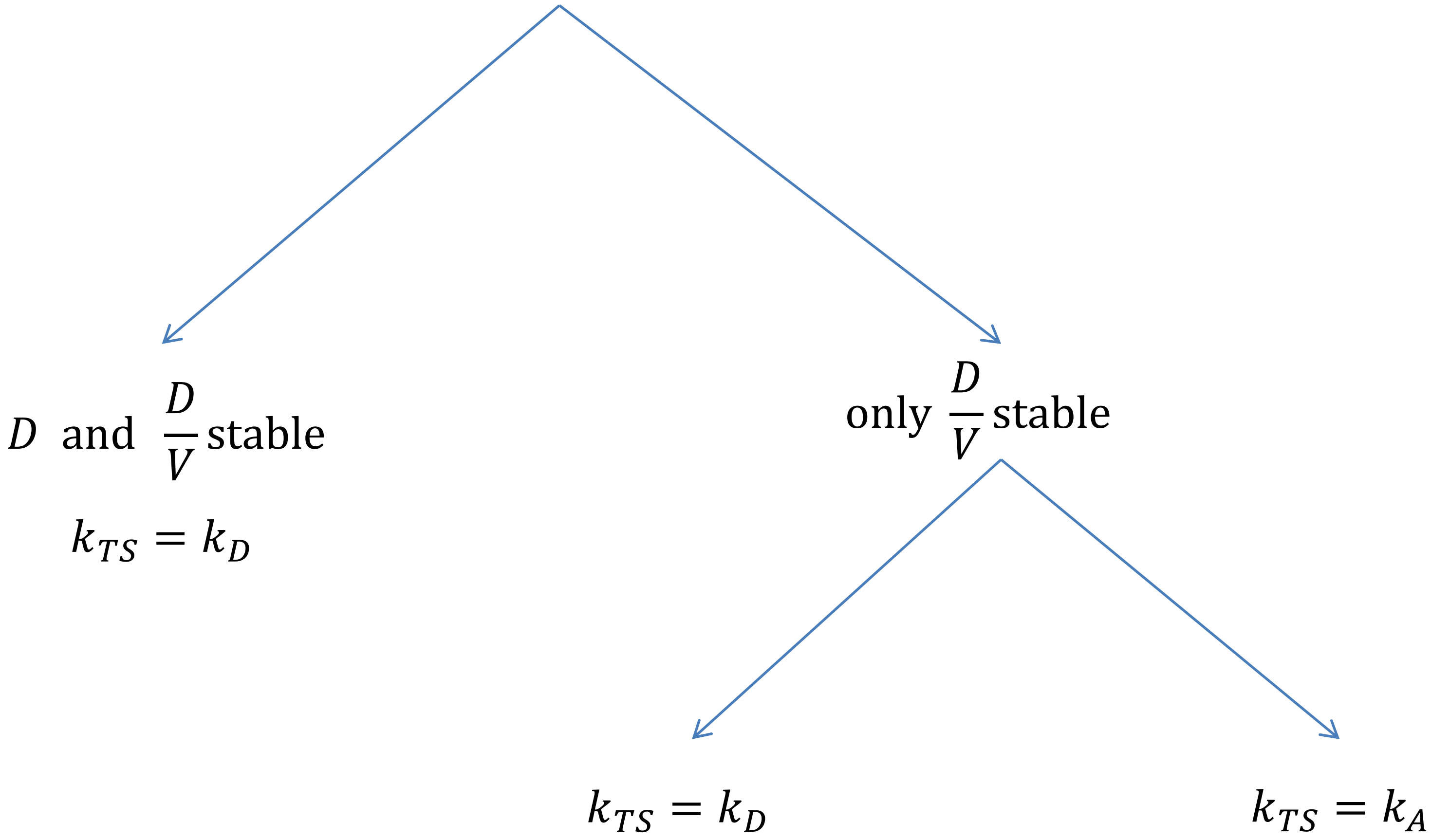
$$V_L \equiv V = V_U + V_{TS} = E + D$$

Also, this should hold

$$\sum_{t=1}^{T\ or\ \infty} \frac{FCF_t^{activity}}{(1+k_a)^t} + \sum_{t=1}^{T\ or\ \infty} \frac{FCF_t^{activity}}{(1+k_{TS})^t} = \sum_{t=1}^{T\ or\ \infty} \frac{FCF_t^{activity}}{(1+WACC)^t}$$

What’s the purpose?*

Different possibilities



D and $\frac{D}{V}$ stable

$k_{TS} = k_D$

only $\frac{D}{V}$ stable

$k_{TS} = k_D$

$k_{TS} = k_A$

WITHOUT TAXES OR OTHER FRICTIONS…

- Understand the original context in which MM developed their groundbreaking contribution to the WACC.
- Understand that, in a “flat World”, it is non-sense to try leveraging the WACC to supposedly reduce it.
- Even if our World is not so flat, these results are important. It means that
  - When there will be more frictions, this equality will explode.
  - When new regulations “flatten” again these frictions (like the NID in Belgium), we should probably be back to a World where the leverage should matter less.

Cost of capital with debt

- CAPM holds
  - Risk-free rate = 5%
  - Market risk premium = 6%
- Consider an all-equity firm:

Market value *V*	100	
Beta	1	
Cost of capital	11%	(=5% + 6% * 1)

- Now consider borrowing 20 to buy back shares.
- Why such a move?
  - Debt is cheaper than equity
  - Replacing equity with debt should reduce the average cost of financing
- What will be the final impact
  - On the value of the company? (Equity + Debt)?
  - On the weighted average cost of capital (WACC)?

Definition of debt and equity contracts

- At some maturity T
  - Debt of face value F
  - Asset of value V_a

	$V_a < F$	$V_a < F$
Debt	V_a	F
Equity	0	$V_a - F$

Before MM (1958) but still for some...

- 2 markets, debt and equity
- Good theory of debt, but no pricing of equity. Use of PE ratio.
- Suppose
  - PE = 10.
  - Debt face value of 4’000 EUR
  - Interest rate is 5%. Yield is 5%.

	No debt (unlevered)	Some debt (levered)
EBIT	1’000	1’000
Interest		200
EBT	1’000	800
Tax (50%)	500	400
Net income	500	400
E	5’000	4’000
D	0	4’000

Modigliani Miller (1958)

- Assume perfect capital markets
  - no taxes/transaction costs
  - no bankruptcy costs
  - no information asymmetry
  - no agency costs (managers maximise NPV)
  - borrowing rate = lending rate
  - capital markets are efficient

and that capital structure does not affect investment.

- Proposition I:
  - The market value of any firm is independent of its capital structure: $V_L = E+D = V_U$
    - ✓ 2 companies with the same cash flows and the same risk have the same value.
- Proposition II:
  - The weighted average cost of capital is independent of its capital structure $WACC = k_{Asset}$
  - k_{Asset} is the cost of capital of an all equity firm

MM 58: Proof by arbitrage

- Value additivity/Fixed pie theory
- Consider two firms (U and L) with identical operating cash flows X
  $V_U = E_U$
  $V_L = E_L + D_L$

	Current cost	Future payoff
Buy **α%** shares of **U**	$\alpha E_U = \alpha V_U$	**αX**
Buy **α%** bonds of **L**	αD_L	$\alpha r D_L$
Buy **α%** shares of **L**	αE_L	$\alpha(X - rD_L)$
Total	$\alpha D_L + \alpha E_L = \alpha V_L$	αX

As the future payoffs are identical, the initial cost should be the same. Otherwise, there would exist an arbitrage opportunity

MM 58: Proof using CAPM

- 1-period company
- *C* = future cash flow, a random variable
- Unlevered company: $V_U = \frac{\mathrm{E}(C) - \lambda \operatorname{cov}(C, R_M)}{1 + r_f}$
- Levered (assume riskless debt): $E = \frac{\mathrm{E}(Div) - \lambda \operatorname{cov}(Div, R_M)}{1 + r_f}$

$$E = \frac{\mathrm{E}[C - (1 + r_f)D] - \lambda \operatorname{cov}([C - (1 + r_f)D], R_M)}{1 + r_f} = \underbrace{\frac{\mathrm{E}(C) - \lambda \operatorname{cov}(C, R_M)}{1 + r_f}}_{=V_U} - D$$

- So: *E* + *D* = V_U

MM 58: Proof using state prices

- 1-period company, risky debt: $V_u > F$ *but* $V_d < F$
- If $V_d < F$, the company goes bankrupt

	Current value	Up	Down
Cash flows	$V_{Unlevered}$	V_u	V_d
Equity	E	$V_u - F$	0
Debt	D	F	V_d

$$V_{Unlevered} = v_u V_u + v_d V_d$$

$$E = v_u \times (V_u - F) + v_d \times 0$$

$$D = v_u \times F + v_d \times V_d$$

$$\begin{aligned} V &= E + D \\ &= [v_u (V_u - F)] + [v_u F + v_d V_d] \\ &= v_u V_u + v_d V_d \\ &= V_{Unlevered} \end{aligned}$$

MM 58: “WACC is independent of leverage”

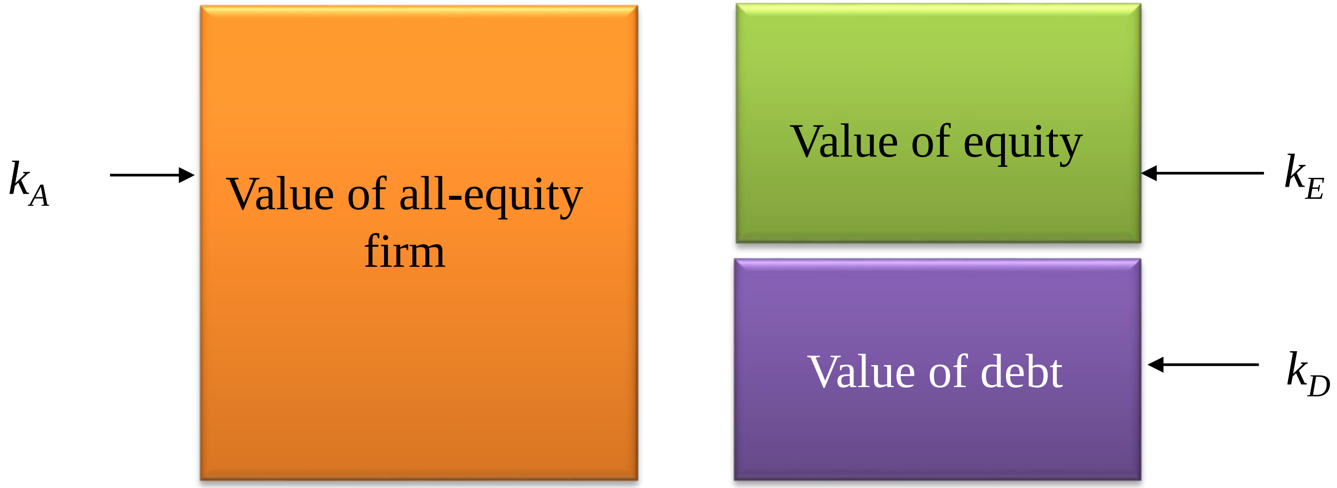
1) $$WACC = \frac{D}{V}k_D + \frac{E}{V}k_E$$

2) $$k_i = r_f + \beta_i\left[E\left(r_m\right) - r_f\right]$$

3) $$\beta_A = \frac{D}{V}\beta_D + \frac{E}{V}\beta_E$$

⇨ $$\begin{aligned} WACC &= \frac{D}{V}\left\{r_f + \beta_D\left[E\left(r_m\right) - r_f\right]\right\} + \frac{E}{V}\left\{r_f + \beta_E\left[E\left(r_m\right) - r_f\right]\right\} \\ &= r_f + \beta_A\left[E\left(r_m\right) - r_f\right] \\ &= k_A \end{aligned}$$

$$V_L \equiv V = V_U = E + D$$



$$k_a = \underbrace{k_e \frac{E}{V} + k_d \frac{D}{V}}_{WACC}$$

Using MM 58

- Value of company: V = 100

	Initial	Final	
Equity	100	80	
Debt	0	20	
Total	100	100	MM I
WACC = r_A	11%	11%	MM II
Cost of debt	-	5%	(assuming risk-free debt)
D/V	0	0.20	
Cost of equity	11%	12.50%	(to obtain *WACC* = 11%)
E/V	100%	80%	

Why are MM I and MM II related?

- Assumption: perpetuities (to simplify the presentation)
- For a levered companies, earnings before interest and taxes will be split between interest payments and dividends payments

$$EBIT = Int + Div$$

- Market value of equity: present value of future dividends discounted at the cost of equity

$$E = Div / k_{Equity}$$

- Market value of debt: present value of future interest discounted at the cost of debt

$$D = Int / k_{Debt}$$

Relationship between firm value and the WACC

- From the definition of the WACC:

$$WACC \times V = k_{Equity} \times E + k_{Debt} \times D$$

- As $k_{Equity} \times E = Div$ and $k_{Debt} \times D = Int$

$$WACC \times V = EBIT$$

$$V = EBIT \;/\; WACC$$

Market value of levered firm

EBIT is independent of leverage

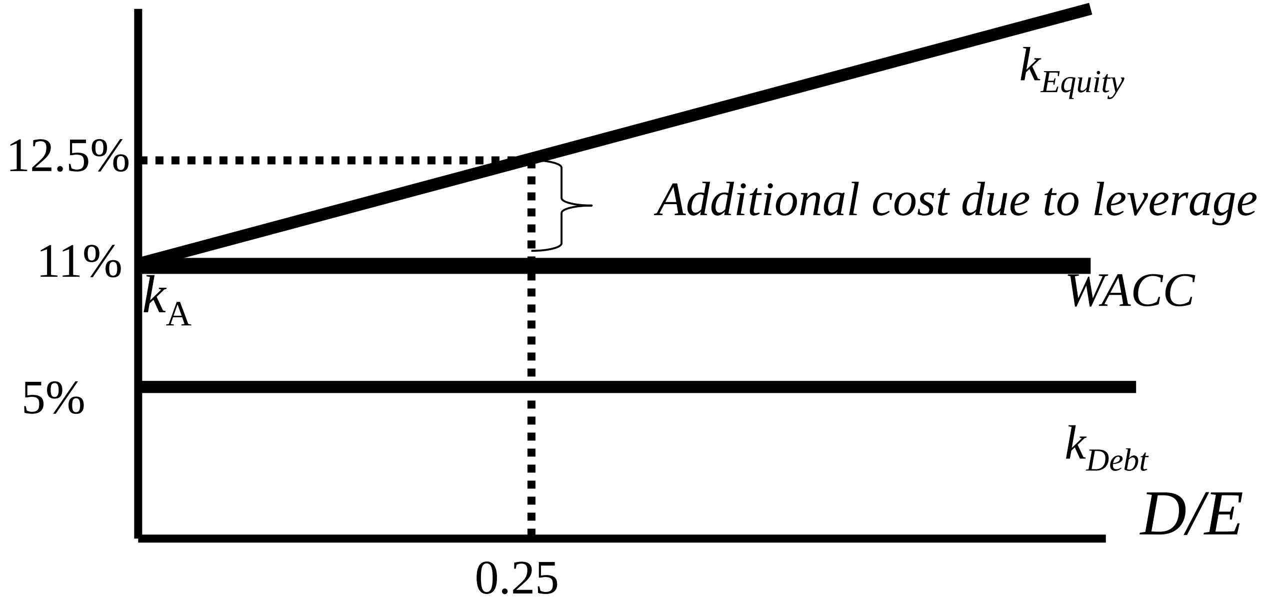
If value of company varies with leverage, so does WACC in opposite direction

MM II: another presentation

- The equality $WACC = k_{Asset}$ can be written as:

$$k_{Equity} = k_{Asset} + (k_{Asset} - k_{Debt}) \times \frac{D}{E}$$

- Expected return on equity is an increasing function of leverage:



MM II: reworking...

Why does k_{Equity} increases with leverage?

- Because leverage increases the risk of equity.
  - » To see this, back to the portfolio with both debt and equity.
  - » Beta of portfolio: $\beta_{Portfolio} = \beta_{Equity} \times X_{Equity} + \beta_{Debt} \times X_{Debt}$
  - » But also: $\beta_{Portfolio} = \beta_{Asset}$
  - » So: $\beta_{Asset} = \beta_{Equity} \times \frac{E}{E+D} + \beta_{Debt} \times \frac{D}{E+D}$
- or $\beta_{Equity} = \beta_{Asset} + (\beta_{Asset} - \beta_{Debt}) \times \frac{D}{E}$

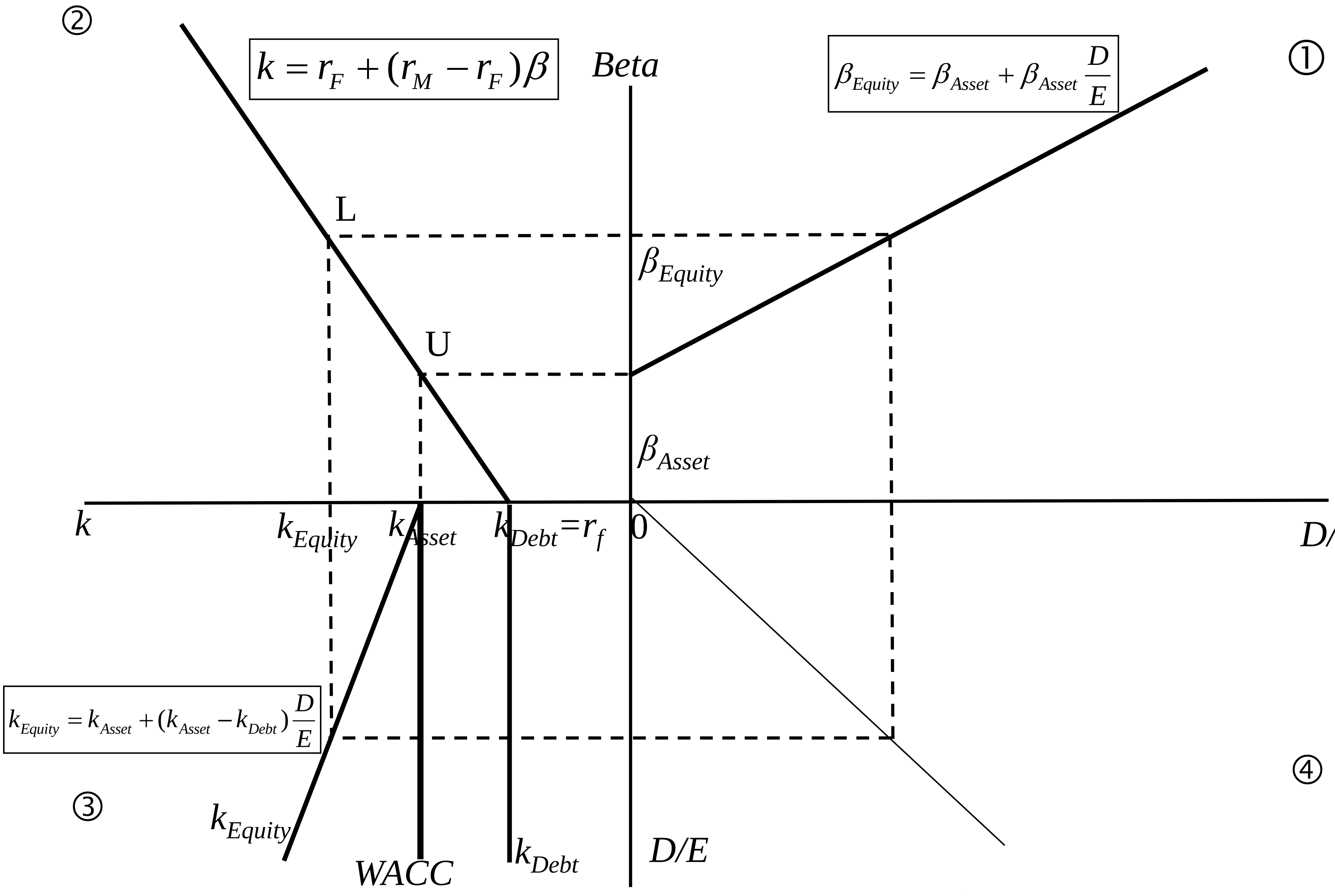
Back to example

- Assume debt is riskless (Hamada's proposition):

$$\beta_{Equity} = \beta_{Asset}\left(1+\frac{D}{E}\right) = \beta_{Asset}\frac{V}{E}$$

- Beta asset = 1
- Beta equity = 1(1+20/80) = 1.25
- Cost of equity = 5% + 6% × 1.25 = 12.50

Summary: the Beta-CAPM diagram



$$k = r_F + (r_M - r_F)\beta$$

$$\beta_{Equity} = \beta_{Asset} + \beta_{Asset}\frac{D}{E}$$

$$k_{Equity} = k_{Asset} + (k_{Asset} - k_{Debt})\frac{D}{E}$$

WITH TAXES OR OTHER FRICTIONS…

- We now introduce taxes, one friction, in the WACC problem (MM63).
  - With taxes, tax deductibility provides a sort of debt for equity «arbitrage».
- We should understand how the expressions presented in the previous set change to integrate tax shields.
- We should also be able to preview some limitations of the WACC as proposed by MM63.
- MM is still the perfect base to extend to more complex issues thereafter.

MMs propositions

- Proposition I
  - Investment consistent with revenues
  - No arbitrage
  - The value of the company should therefore be independent of the leverage
  - Valuing investments can be done irrespective of financing
- Proposition II
  - Market feedback exists.
  - If I holds, knowing that equity is riskier than debt, equity cost should be higher, even if there is no bankruptcy event made possible.
  - If I holds, it means that the same result can be obtained whatever is the WACC.
  - A WACC independent from leverage would mean: there exists an adjustable cost of equity.

MM (1963) with taxes: Corporate Tax Shield

- Interest payments are tax deductible ➔ tax shield
- Tax shield = Interest payment × Corporate Tax Rate

$$k_D \times D \times t_c$$

  - k_D : cost of new debt
  - D : market value of debt
  - Value of levered firm

    = Value if all-equity-financed + PV(Tax Shield)

- PV(Tax Shield) - Assume permanent borrowing

$$PV(Tax\,Shield) = \frac{t_C \times k_D D}{k_D} = t_C D$$

- Other assumptions?
- Value of the firm: $V_L = V_U + t_c D$

Example

	U	L
<u>Balance Sheet</u>		
Total Assets	1,000	1,000
Book Equity	1,000	500
Debt (8%)	0	500
<u>Income Statement</u>		
EBIT	200	200
Interest	0	40
Taxable Income	200	160
Taxes (40%)	80	64
Net Income	120	96
Dividend	120	96
Interest	0	40
Total	120	136

Adjusted Present Value approach (APV)

Assume k_A= 10%, k_D = 5%

(1) Value of all-equity-firm:

V_U = 120 / 0.10 = 1,200

(2) PV(Tax Shield):

Tax Shield = 40 x 0.40 = 16

PV(TaxShield) = 16/0.05 = 320

(3) Value of levered company:

V_L = 1,200 + 320 = 1,520

(4) Market value of equity:

D = 40/.05 = 800

$E_L = V_L - D$ = 1,520 - 800 = 720

What about cost of equity?

1. Cost of equity increases with leverage:

$$k_E = k_A + (k_A - k_D) \times (1 - t_C) \times \frac{D}{E}$$

Proof:

$$E = \frac{(EBIT - k_D D) \times (1 - t_C)}{k_E}$$

But $V_U = EBIT(1\text{-}t_C)/k_A$

and $E = V_U + t_C D - D$

Replace and solve

2. Beta of equity increases

$$\beta_E = \beta_A \left[1 + (1 - t_C) \frac{D}{E} \right]$$

In example:

k_E = 10% +(10%-5%)(1-0.4)(800/720)

= 13.33%

or

$k_E = DIV/E$ = 96/720 = 13.33%

What about the WACC?

- Weighted average cost of capital: discount rate used to calculate the market value of a firm by discounting net operating profit less adjusted taxes
  - $NOPLAT = EBIT(1\text{-}t_c)$
  - $V_L = EBIT(1\text{-}t_c) / WACC$

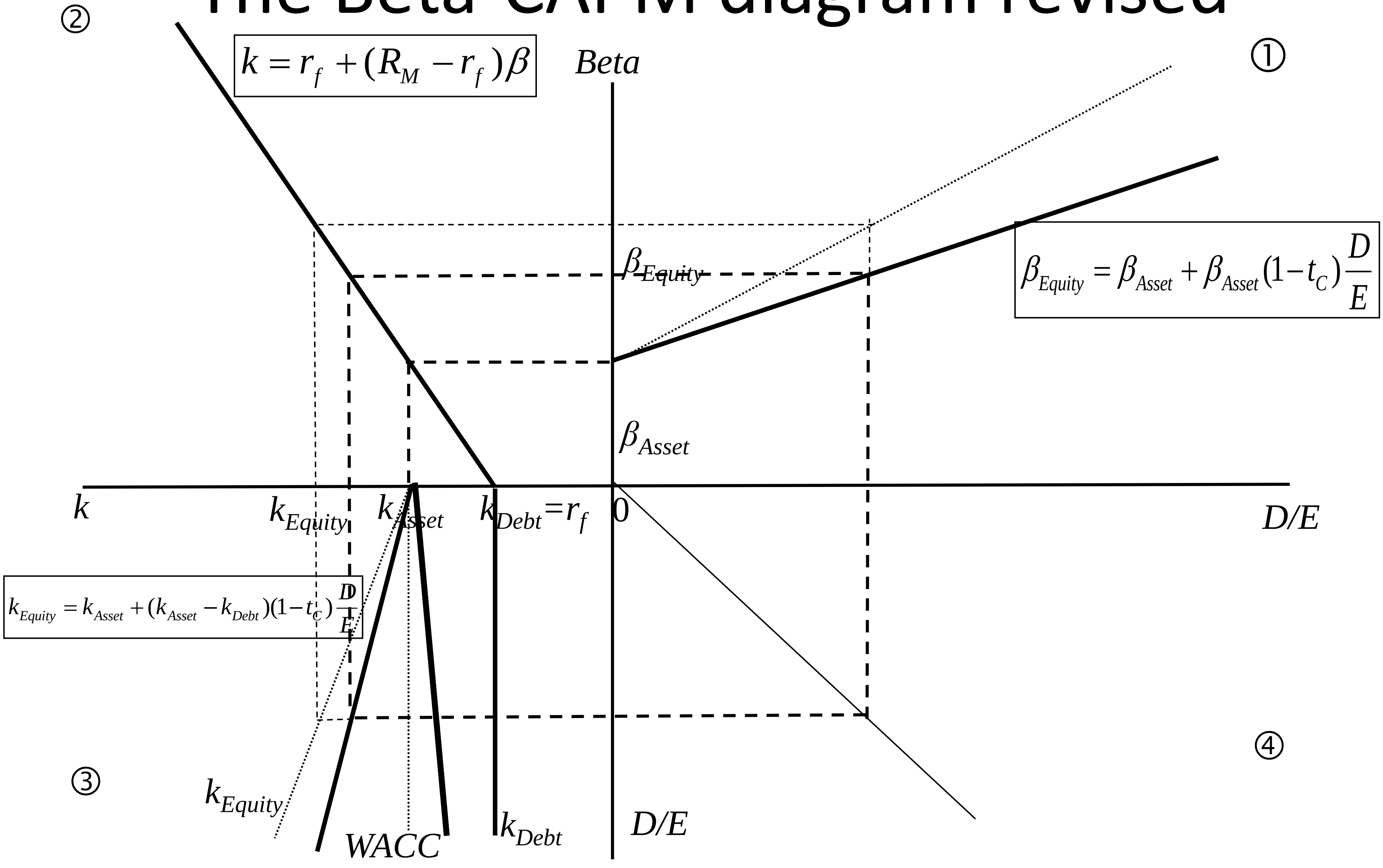
- As: $V_L > V_U$ $WACC < k_A$

$$k_E E + k_D(1-t_C)D = EBIT(1-t_C)$$

$$WACC = k_E \times \frac{E}{V_L} + k_D(1-t_C) \times \frac{D}{V_L}$$

In example: $NOPLAT = 120$
$V = 1{,}520$
$WACC = 13.33\% \text{ x } 0.47 + 5\% \text{ x } 0.60 \text{ x } 0.53 = 7.89\%$

The Beta-CAPM diagram revised

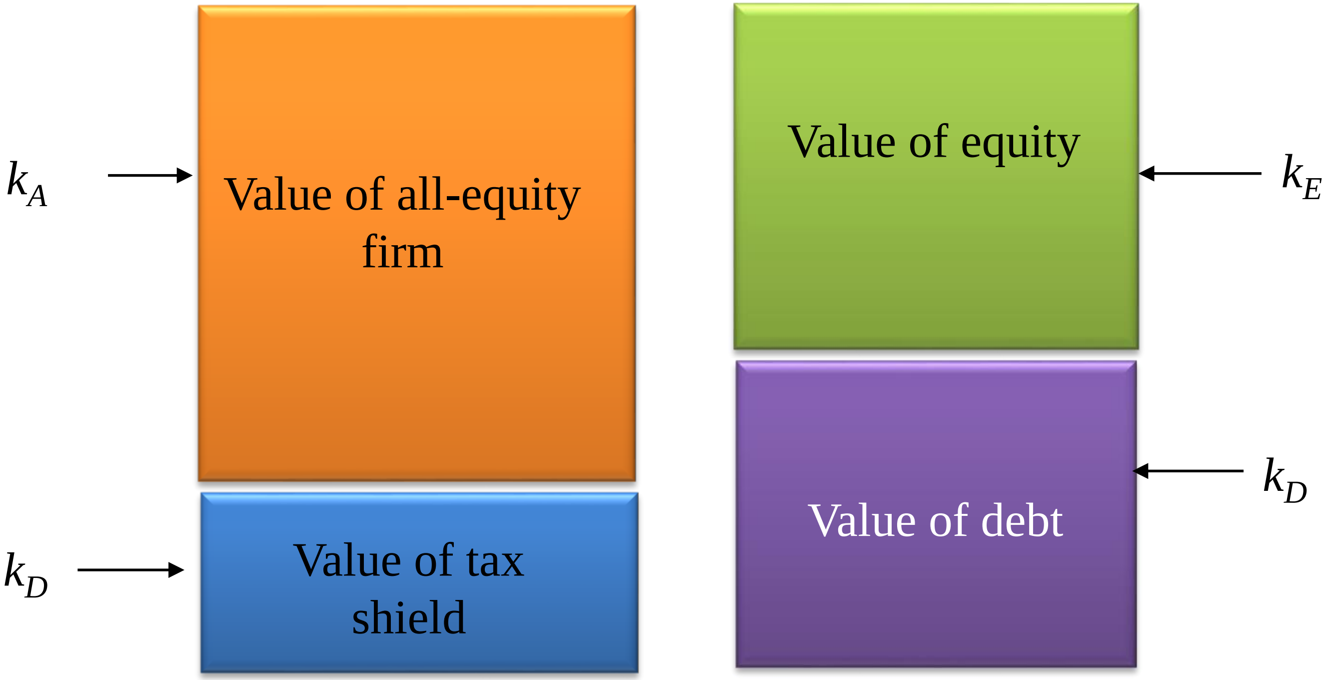


$$k = r_f + (R_M - r_f)\beta$$

$$\beta_{Equity} = \beta_{Asset} + \beta_{Asset}(1-t_C)\frac{D}{E}$$

$$k_{Equity} = k_{Asset} + (k_{Asset} - k_{Debt})(1-t_C)\frac{D}{E}$$

$$V_L \equiv V = V_U + V_{TS} = V_U + t_C D = E + D$$



k_A → Value of all-equity firm

k_D → Value of tax shield

Value of equity ← k_E

Value of debt ← k_D

$$k_a \frac{V_U}{V_L} + kd \frac{t_C D}{V_L} = k_e \frac{E}{V_L} + k_d \frac{D}{V_L}$$

WACC – Modigliani Miller formula

$$k_A \frac{V_U}{V_L} + k_D \frac{t_C D}{V_L} = k_E \frac{E}{V_L} + k_D \frac{D}{V_L}$$

$$k_A \frac{V_L - t_C D}{V_L} = k_E \frac{E}{V_L} + k_D (1 - t_C) \frac{D}{V_L}$$

$$WACC \equiv k_E \frac{E}{V_L} + k_D (1 - t_C) \frac{D}{V_L}$$

$$\Rightarrow \quad WACC = k_A \left(1 - t_C \frac{D}{V_L} \right)$$

WACC – using Modigliani-Miller formula

- Assumptions:
  - 1. Perpetuity
  - 2. Debt constant
  - 3. D/V = L

$$V_L = \frac{EBIT(1-t_C)}{WACC}$$

Proof:

- Market value of unlevered firm:

$V_U = EBIT\,(1\text{-}t_c)/k_{Asset}$

- Market value of levered firm:

$V_L = V_U + t_C\,D$

$$V_L = \frac{EBIT(1-t_C)}{k_A} + t_C \frac{D}{V_L} V_L$$

- Define: $L \equiv D/V_L$
- Solve for V_L:

$$V_L = \frac{EBIT(1-t_C)}{k_A(1-t_C L)} = \frac{EBIT(1-t_C)}{WACC}$$

MM formula: example

Data

Investment	100
Pre-tax CF	22.50
k_A	9%
k_D	5%
t_C	40%

Base case NPV: -100 + 22.5(1-0.40)/.09 = 50

Financing:
Borrow 50% of PV of future cash flows after taxes
$D = 0.50\ V$

Using MM formula: $WACC = 9\%(1-0.40 \times 0.50) = 7.2\%$

NPV = -100 + 22.5(1-0.40)/.072 = 87.50

Same as APV introduced previously? To see this, first calculate *D.*
As: $V_L = V_U + t_C D = 150 + 0.40\ D$
and: $D = 0.50\ V$
$V = 150 + 0.40 \times 0.50 \times V \rightarrow V = 187.5 \rightarrow D = 93.50$
$\rightarrow APV = NPV_0 + T_C D = 50 + 0.40 \times 93.50 = 87.50$

Using the standard WACC formula

- Step 1: calculate k_E using $$k_E = k_A + (k_A - k_D)(1 - t_C)\frac{D}{E}$$
  - As D/V = 0.50, D/E = 1
  - k_E = 9% + (9% - 5%)(1-0.40)(0.50/(1-0.50)) = 11.4%
- Step 2: use standard WACC formula $$WACC = k_E \frac{E}{V} + k_D(1 - t_C)\frac{D}{V}$$
- WACC = 11.4% x 0.50 + 5% x (1– 0.40) x 0.50 = 7.2%

Same value as with MM formula

Adjusting WACC for debt ratio or business risk

- Or (assuming debt is riskless):

- Step 1: unlever the WACC

$$k_A\left(1-t_C\frac{D}{V}\right)=k_E\frac{E}{V}+k_D\frac{D}{V}$$

- Step 2: Estimate cost of debt at new debt ratio and calculate cost of equity

$$k_E=k_A+(k_A-k_D)(1-t_C)\frac{D}{E}$$

- Step 3: Recalculate WACC at new financing weights

- Step 1: Unlever beta of equity

$$\beta_{equity}=\beta_{asset}(1+(1-T_C)\frac{D}{E})$$

- Step 2: Relever beta of equity and calculate cost of equity

- Step 3: Recalculate WACC at new financing weights

Debt not permanent

	0	1	2	3	4	5	6
EBITDA		340	340	340	340	340	340
Dep		100	100	100	100	100	100
EBIT		240	240	240	240	240	240
Interest		40	32	24	16	8	0
Taxes		80	83	86	90	93	96
Earnings		120	125	130	134	139	144
CFop		220	225	230	234	239	277
CFinv		-100	-100	-100	-100	-100	-100
DIV		-20	-25	-30	-34	-39	-144
ΔDebt		-100	-100	-100	-100	-100	
Book eq.	500	600	700	800	900	1,000	1,000
Debt	500	400	300	200	100	0	0

Valuation of the company

- Assumptions: k_A = 10%, k_D = 4%

1. Value of unlevered company
    - ✓ As Unlevered Free Cash Flow = 144, V_U = FCFU / k_A = 1,440
2. Value of tax shield (discounted with k_D)
    - ✓ $$V_{TS} = \frac{16}{1.04} + \frac{12.8}{(1.04)^2} + \frac{9.6}{(1.04)^3} + \frac{6.4}{(1.04)^4} + \frac{3.2}{(1.04)^5} = 44$$
3. Value of levered company
    - ✓ V = 1,440 + 44 = 1,484
4. Value of debt
    - ✓ $$D = \frac{40+100}{1.04} + \frac{32+100}{(1.04)^2} + \frac{24+100}{(1.04)^3} + \frac{16+100}{(1.04)^4} + \frac{8+100}{(1.04)^5} = 555$$
5. Value of equity
    - ✓ E = 1,484 - 555 = 980

The leverage puzzle

- Implications of MM (1963)
  - Optimal capital structure is 100% debt
  - Debt is good
  - Leverage creates tax shield
  - Tax arbitrage. LBOs. Strip financing.
- Therefore:
  - If $V_L > V_U$, companies should borrow as much as possible to reduce their taxes.
  - But observed leverage ratios are fairly low
    - ✓ For the US, median D/V ≈ 23%
  - Assume $t_C = 40\%$
  - Value of tax shield = $t_C D$
  - Median $V_{TS} \approx 9\%$
  - Why don't companies borrow more?

Corporate and Personal Taxes

- Debt and equity face differential taxation at personal level.
  - Investors who are in higher tax brackets require higher rates of return on corporate debt to compensate for their tax disadvantage.
- Suppose operating income = 1
- If paid out as

	Interest	Equity income
Corporate tax	0	t_C
Income after corporate tax	1	$1-t_C$
Personal tax	t_P	$t_{PE}(1-t_C)$
Income after all taxes	$1-t_P$	$(1-t_{\mathrm{PE}})(1-t_C)$

$$t_{PE}=\alpha t_G+(1-\alpha)t_P$$

V_{TS} with corporate and personal taxes (Miller 1977)

- Net cash flows to shareholders: $\left(EBIT - k_D D\right)\left(1-t_C\right)\left(1-t_{PE}\right)$
- Net cash flows to debtholders: $\left(k_D D\right)\left(1-t_P\right)$
- Net cash flows to debt + equity: $EBIT\left(1-t_C\right)\left(1-t_{PE}\right)+k_D D\left[\left(1-t_P\right)-\left(1-t_C\right)\left(1-t_{PE}\right)\right]$
- So we have: $$V_L = V_U + \underbrace{\left[1-\frac{(1-t_C)(1-t_{PE})}{(1-t_P)}\right]D}_{\text{value of tax shields}}$$
- Tax advantage of debt is positive if: $1 - t_P > (1 - t_c)(1 - t_{PE})$
- Note
  - if $t_P = t_{PE}$, then $V_{TS} = t_c D$
  - Or if all equity return paid as dividend

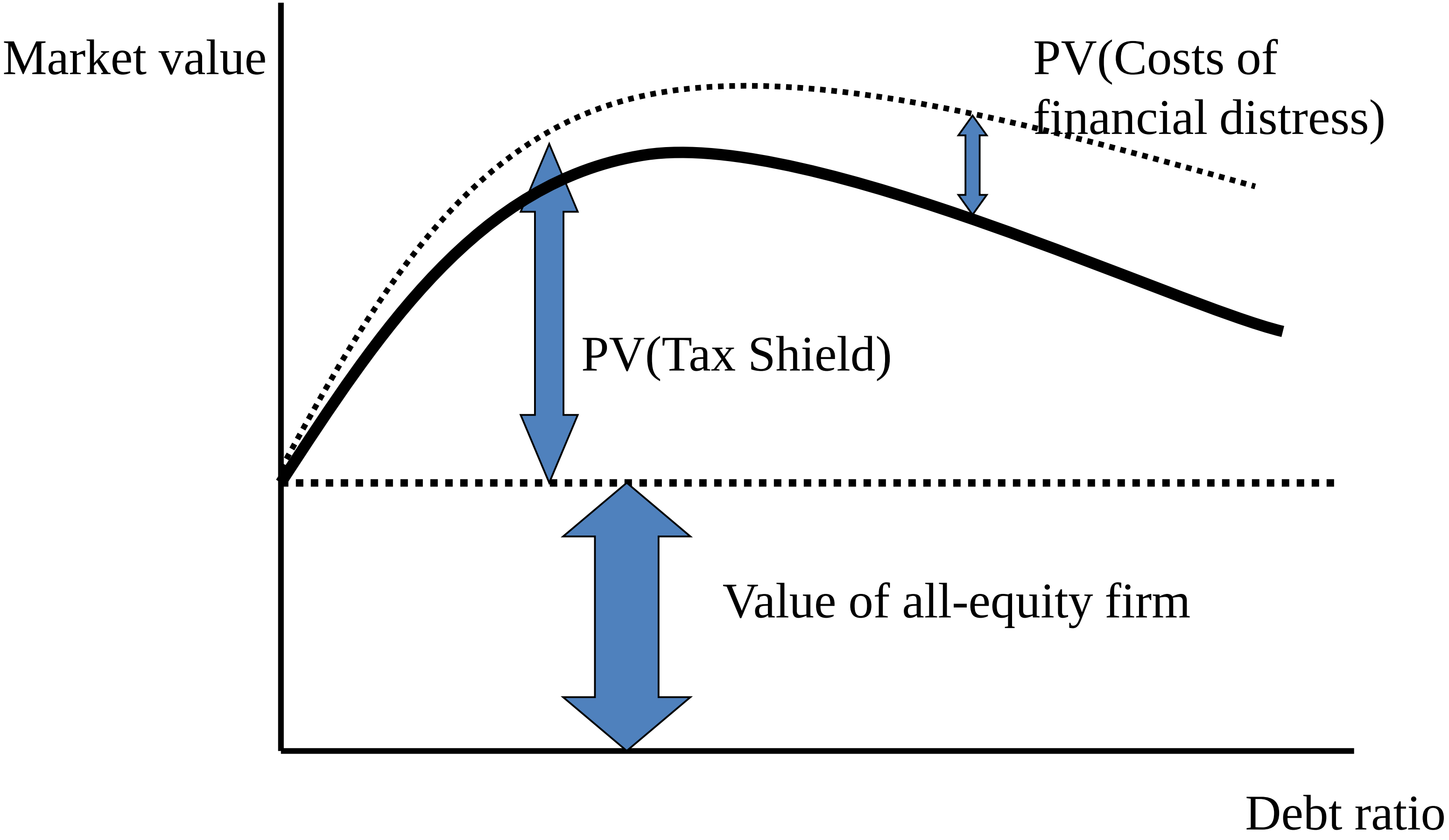
Miller (1977): Reasons

- Why could we have $t_P > t_{PE}$?
  - Capital gains tax rate < interest income tax rate
  - Defer capital gains tax
  - Gains and losses in well diversified portfolios tend to offset each other
  - 80% of the dividends that a taxable company receives can be excluded from the taxable income
  - Many types of investment funds pay no taxes at all
- Assuming $t_{PE} = 0$, tax advantage if $t_C > t_P$. It is the marginal investor who matters.
  - Miller equilibrium: the aggregate economy-wide D/E ratio is such that $t_C > t_P$. No individual firm has an optimal D/E ratio.
- Does this makes sense?
  - Tax benefits are probably less than $t_C D$ (De Angelo and Masulis)
  - But tax benefits are greater than 0, especially post 1986 (in US)
  - There are cross-sectional differences in effective t_C since firms may not be able to use all tax shields. Thus the theory should have some ability to explain leverage.

Empirical evidence → there is a puzzle...

- On tax shields in general
  - Firms have debt ratios much lower than 100%
- On corporate and personal taxes
  - None if you run D/E on tax rates.
  - But capital structure tends to be sticky. It is not always an optimum as this static model suggests. MacKie-Mason find evidence on the role of taxes based on marginal financing choices.
- If V_{TS} >0, why not 100% debt?
  - cost of financial distress
    - ✓ As debt increases, probability of financial problem increases
    - ✓ The extreme case is bankruptcy.
    - ✓ Financial distress might be costly
  - Leads to the static trade-off theory
    - ✓ *L* = costs of financial distress $V_L = V_U + t_C D - L$
  - Directs costs: lawyers, bankers, management time
  - Indirect costs: reputational cost, loss of confidence, disruptions, etc...

Trade-off theory



There is still a puzzle...

- Warner (1977): “are distress costs big enough to explain the low leverage of many firms?”
    - 1% of the market value of the firm 7 years before bankruptcy
    - 5.32% of the market value of the firm immediately before bankruptcy

      These costs must be multiplied by *P*(bankruptcy) to obtain the expected cost of bankruptcy (below 10% in general) → very low compared to firm value.
- Also:
    - Wide variations in leverage of firms with similar operating risk.
    - In the US, D/E ratios in the 1920s were similar to ratios in the 1950s despite a large increase in t_C from 10% to 52%.
    - Some companies hold debt even with $t_C = 0$.
- Therefore:
    - What limits debt use by firms given small estimates of “direct” bankruptcy costs?
    - Why might firms use debt even with no tax advantage of debt?

THE TERM «APV» …

- The term «APV» stands for « Adjusted Present Value».
- It is mainly a term brought by people as Timothy Luerhman from HBS in the late 90s to advocate for an analysis not based on the WACC, but based on the explicit valuation of all financial side effects aside of the the NPV of the activity itself.
  - Luehrman (1997), "Using APV: A Better Tool for Valuing Operations Harvard Business Review", May-June 1997
- As such, it is just an application of the equality brought by MM, allowing for more specificities (than in the simple MM case) to be precisely computed.

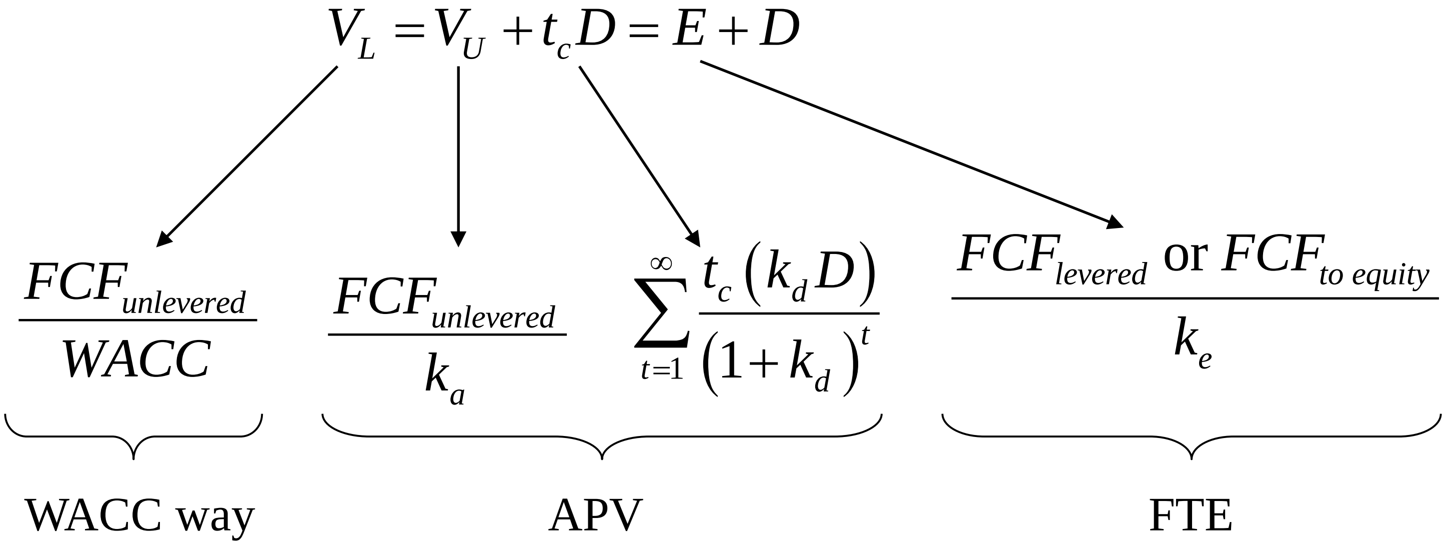
- We will use the excuse of this subsection to compare
  - The WACC approach
  - The APV approach
  - The FTE approach

Capital budgeting and Financing

- Projects or Firms capital budgeting decisions can be affected by many financing side-effects:
  - Interest tax shields
  - Transaction costs
  - Flotation costs
  - Subsidies
  - …
- There are two main standard tracks to run a DCF analysis on a project or firm with financing side-effects:
  1. The standard NPV approach with a WACC that is adjusted to take implicitly into account the impact of the financing decision
     - NPV using an adapted WACC
  2. The Adjusted Present Value: we just discount explicitly every group of cash flows at its corresponding rate.
     - APV = Base case NPV + NPV(financing effects)

Basis of reasoning

- Do you remember this expression?
(remember also its assumptions!)



$$V_L = V_U + t_c D = E + D$$

$$\underbrace{\frac{FCF_{unlevered}}{WACC}}_{\text{WACC way}} \qquad \underbrace{\frac{FCF_{unlevered}}{k_a} \qquad \sum_{t=1}^{\infty} \frac{t_c\left(k_d D\right)}{\left(1+k_d\right)^t}}_{\text{APV}} \qquad \underbrace{\frac{FCF_{levered} \text{ or } FCF_{to\ equity}}{k_e}}_{\text{FTE}}$$

- Three methodologies that should be consistent under certain assumptions and context!
  - Simple context: everything can be summarized in a rate
  - Perpetuity! → there is a single WACC (à priori) while we can discount any series of CFs quite explicitly with any specific value each period.

The three methods compared for a project

(assuming perpetual cash flows)

- WACC

$$\sum_{t=1}^{\infty} \frac{FCF_t^{unlevered}}{(1+WACC)^t} - I$$

- APV

$$\sum_{t=1}^{\infty} \frac{FCF_t^{unlevered}}{(1+k_a)^t} + PV(\text{financing effects}) - I$$

- FTE

$$\sum_{t=1}^{\infty} \frac{FCF_t^{levered}}{(1+k_e)^t} - (I - D)$$

1 - Bicksler Enterprises (RWJ p. 487)

- Settings of the Bicksler project:
  - Investment: 10 mio
  - Maturity: 5 years
  - Straight-line depreciation
  - Revenues less cash expense : 3.5 mio/year
  - Corporate tax rate: 34%
  - $r_f = k_d = 10\%$
  - $k_a = 20\%$
- All-equity value ?

From Ross, Westerfield & Jaffe, 7^{e} edition, p. 487

Adding debt

- Settings for the debt issue:
    - Debt issue obtainable: non-amortizing loan of 7.5 mio after flotation costs
    - Maturity: 5 years
    - $r_f = k_d = 10\%$
    - Flotation costs: 1%
- Debt issue
- Net Flotation Costs

From Ross, Westerfield & Jaffe, 7^{e} edition, p. 487

Adding debt (bis)

- Tax Shields (prior development)

From Ross, Westerfield & Jaffe, 7^{e} edition, p. 487

Adding debt (ter)

- Tax Shields (result)

- APV result:

From Ross, Westerfield & Jaffe, 7e edition, p. 487

Non-market-rate financing (subsidies,…)

- Settings for the debt issue:
  - Debt issue obtainable: The State of New Jersey grants a non-amortizing loan of 7.5 mio at 8% with flotation costs absorbed by the State.
  - Maturity: 5 years

- NPV subsidized debt:

- APV result:

From Ross, Westerfield & Jaffe, 7e edition, p. 487

Decomposition of the subsidy

From Ross, Westerfield & Jaffe, 7^{e} edition, p. 487

2 - Alternative example

- Endowments
  - Cost of investment 10,000
  - Incremental earnings 1,800 / year
  - Duration 10 years
  - Discount rate rA 12%

- **Base-case NPV** = -10,000 + 1,800 x a_{10} = 170

1. **Stock issue**
   - Issue cost : 5% from gross proceed
   - Size of issue : 10,526 (= 10,000 / (1-5%))
   - Issue cost = 526
   - **APV** = + 170 - 526 = - 356

Borrowing?

2. Borrowing

- Suppose now that 5,000 are borrowed to finance partly the project
- Cost of borrowing : 8%
- Constant annuity: 1,252/year for 5 years
- Corporate tax rate = 40%

Year	Balance	Interest	Principal	Tax Shield
1	5,000	400	852	160
2	4,148	332	920	133
3	3,227	258	994	103
4	2,223	179	1,074	72
5	1,160	93	1,160	37

- PV(Tax Shield) = 422
- **APV** = 170 + 422 = 592

Discounting Safe, Nominal Cash Flows

"The correct discount rate for safe, nominal cash flows is your company's after-tax, unsubsidized borrowing rate" (Brealey and Myers, Chap19 – 19.5)

- Discounting
  - after-tax cash flows
  - at an after-tax borrowing rate $k_D(1-t_c)$

leads to the equivalent loan (the amount borrowed through normal channels)

- Examples:
  - Payout fixed by contract
  - Depreciation tax shield
  - Financial lease

APV calculation with subsidized borrowing

3. **Subsidized borrowing**
   - Suppose now that you have an opportunity to borrow at 5% when the market rate is 8%.
   - What is the NPV resulting from this lower borrowing cost?
     1. Compute after taxes cash flows from borrowing
     2. Discount at cost of debt after taxes
     3. Subtract from amount borrowed

- The approach developed in this section is also applicable for the analysis of leasing contracts (See B&M Chap 25)

Subsidized loan

- To understand the procedure, let's start with a very simple setting:
  - 1 period, certainty
  - Cash flows after taxes: $C_0 = -100$ $C_1 = +105$
  - Corporate tax rate: 40%, $k_A = k_D = 8\%$

- **Base case**: $NPV_0 = -100 + 105/1.08 = -2.78 < 0$

- **Debt financing at market rate** (8%)
  - PV(Tax Shield) = (0.40)(8) / 1.08 = 2.96
  - APV = - 2.78 + 2.96 = 0.18 > 0

NPV of subsidized loan

- **Debt financing at subsidized rate**
  - » You can borrow 100 at 5% (below market borrowing rate -8%)
  - » What is the NPV of this interest subsidy?
  - » Net cash flow with subsidy at time t=1: -105 + 0.40 × 5 = -103
  - » How much could I borrow without subsidy for the same future net cash flow?
    - ✓ Solve: B + 8% B - 0.40 × 8% × B = 103
    - ✓ Solution:

Net cash flow

$$B = \frac{103}{1+8\%(1-0.40)} = \frac{103}{1.048} = 98.28$$

After-tax interest rate

- ✓ NPVsubsidy = +100 – 98.28 = $\boxed{1.72}$

PV(Interest Saving)
=(8 – 5)/1.048 = 2.86

+

PV(ΔTaxShield)
=0.40(5 – 8)/1.048 = -1.14

3 - APV calculation

NPV base case	NPV0	= - 2.78
PV(Tax Shield) no subsidy	PV(TaxShield)	= 2.96
NPV interest subsidy	NPVsubsidy	= 1.72
Adjusted NPV	APV	= 1.90

- Check — After tax cash flows

	t = 0	t = 1
Project	- 100	+ 105
Subsidized loan	+100	- 103
Net cash flow	0	+ 2

- How much could borrow today against this future cash flow?
  - » X + 8% X - (0.40)(8%) X = 2 → X = 2/1.048 = 1.90

A formal proof

- Notation
  - » C_t : net cash flow for subsidized loan
  - » r : market rate
  - » D : amount borrowed with interest subsidy
  - » B_0 : amount borrowed without interest subsidy to produce identical future net cash flows
  - » B_t : remaining balance at the end of year t
  - » For final year T: $C_T = B_{T-1} + k(1-t_c)\, B_{T-1}$ (final reimbursement + interest after taxes)
  - » 1 year before: $C_{T-1} = (B_{T-2} - B_{T-1}) + k(1-t_c)\, B_{T-2}$ (partial reimbursement + interest after taxes)

$$\boxed{B_{T-1} = \frac{C_T}{1+k(1-t_C)}}$$

$$\boxed{B_{T-2} = \frac{C_{T-1}}{1+k(1-t_C)} + \frac{C_T}{[1+k(1-t_C)]^2}}$$

- At time 0: $$\boxed{B_0 = \sum_{t=1}^{T} \frac{C_t}{[1+k(1-t_C)]^t}}$$
- $NPV_{subsidy} = D - B_0$

Back to initial example

Data	
Market rate	8%
Amount borrowed	5,000
Borrowing rate	5%
Maturity	5 years
Tax rate	40%
Annuity	1,155

Net Cash Flows Calculation

Year	Balance	Interest	Repayment	Tax Shield	Net CF
1	5,000	250	905	100	1,055
2	4,095	205	950	82	1,073
3	3,145	157	998	63	1,092
4	2,147	107	1,048	43	1,112
5	1,100	55	1,100	22	1,133

B_0 = PV(NetCashFlows) @ 4.80% = 4,750
$NPV_{subsidy}$ = 5,000 - 4,750 = + 250

APV calculation:

NPV base case	NPV_0	= + 170
PV Tax Shield without subsidy	PV(TaxShield)	= + 422
NPV Subsidy	$NPV_{subsidy}$	= + 250
APV		= + 842

«NEW» DEVELOPMENTS…

- The Value of the company is not supposed to remain constant over time; therefore we cannot assume D/V constant and D constant, which is what MM assume in their framework.
  - So we must refine our formulations and use «time subscripts» in many variables we are using…
- Understand why the standard WACC (MM63) is not robust through time
- Apply other developments to the WACC formula to obtain versions of the WACC more sustainable with the idea of a growing on-going concern

How to value a levered company? (base reasoning)

- Value of levered company: $V_L \equiv V = V_U + V_{TS} = E + D$
- In general, WACC changes over time :

$$FCF_t + k_D t_C D_{t-1} + V_{L,t} = V_{L,t-1}\left(1 + k_{E,t}\frac{E_{t-1}}{V_{L,t-1}} + k_D \frac{D_{t-1}}{V_{L,t-1}}\right)$$

Expected payoff =
Free cash flow unlevered
+ Interest Tax Shield
+ Expected value

Expected return for debt and
equity investors

Rearrange:

$$FCF_t + V_{L,t} = V_{L,t-1}\left(1 + k_{E,t}\frac{E_{t-1}}{V_{L,t-1}} + k_D(1-t_C)\frac{D_{t-1}}{V_{L,t-1}}\right)$$

$$= V_{L,t-1}(1 + WACC_t)$$

Solve:

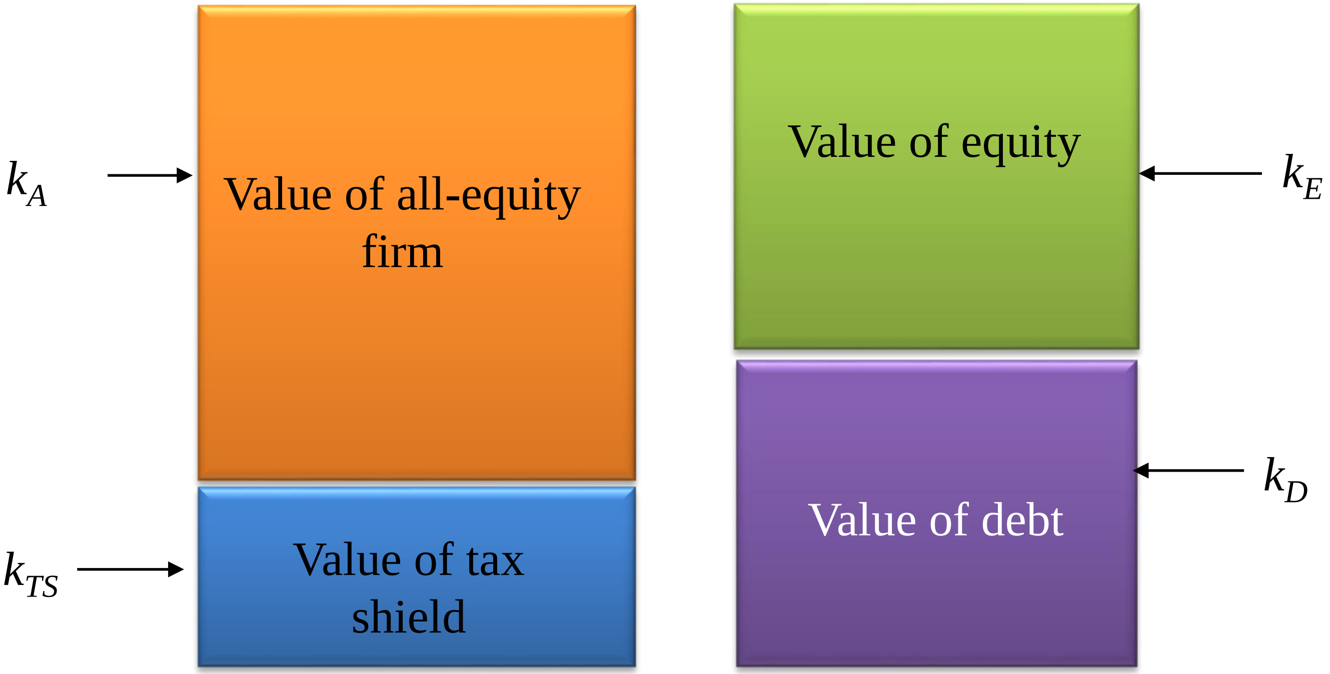
$$V_{L,t-1} = \frac{FCF_t + V_{L,t}}{1 + WACC_t}$$

Comments

- In general, the WACC changes over time. But to be useful, we should have a constant WACC to use as the discount rate. This can be obtained by restricting the financing policy.
- 2 possible financing rules:
  - Rule 1: Debt fixed → Borrow a fraction of initial project value
    - ✓ Interest tax shields are constant. They are discounted at the cost of debt.
  - Rule 2: Debt rebalanced → Adjust the debt in each future period to keep it at a constant fraction of future project value.
    - ✓ Interest tax shields vary. They are discounted at the opportunity cost of capital (except, possibly, for next tax shield –cf Miles and Ezzel)

A general framework

$$V_L \equiv V = V_U + V_{TS} = E + D$$



k_A

Value of all-equity firm

k_{TS}

Value of tax shield

Value of equity

k_E

Value of debt

k_D

$$k_A \frac{V_U}{V_L} + k_{TS} \frac{V_{TS}}{V_L} = k_E \frac{E}{V_L} + k_D \frac{D}{V_L}$$

Cost of equity calculation

$$k_A \frac{V_L - V_{TS}}{V_L} + k_{TS} \frac{V_{TS}}{V_L} = k_E \frac{E}{V_L} + k_D \frac{D}{V_L}$$

$$k_A (V_L - V_{TS}) + k_{TS} V_{TS} = k_E E + k_D D$$

$$\rightarrow \quad k_E = k_A + (k_A - k_D)\frac{D}{E} - (k_A - k_{TS})\frac{V_{TS}}{E}$$

If $k_{TS} = k_D$ (MM) and $V_{TS} = t_C D$:

$$k_E = k_A + (k_A - k_D)(1 - t_C)\frac{D}{E}$$

Similar formulas for beta equity (replace *k* by β)

WACC

$$WACC = k_E \frac{E}{V_L} + k_D(1-t_C)\frac{D}{V_L}$$

$$= \left[k_A + (k_A - k_D)\frac{D}{E} - (k_A - k_{TS})\frac{V_{TS}}{E} \right]\frac{E}{V_L} + k_D(1-t_C)\frac{D}{V_L}$$

➔ $$WACC = k_A\left(1-\frac{V_{TS}}{V_L}\right) - k_D t_C \frac{D}{V_L} + k_{TS}\frac{V_{TS}}{V_L}$$

If $k_{TS} = k_D$ and $V_{TS} = t_C D$ (MM) :

$$WACC = k_A \frac{V_U}{V_L} = k_A\left(1 - t_C\frac{D}{V_L}\right)$$

Rule 1: Debt fixed (Modigliani Miller)

- Assumption:
  - constant perpetuities $FCF_t = EBIT(1\text{-}t_C) = k_A\, V_U$
  - D constant.
- Define: $L = D/V_L \equiv D/V$

$$V_L = \frac{EBIT(1-t_C)}{k_A} + t_C L V_L \Rightarrow V_L = \frac{EBIT(1-t_C)}{k_A - k_A t_C L}$$

$$V_{TS} = t_C D = t_C L V_L$$

➔ $$k_E = k_A + (k_A - k_D)(1-t_C)\frac{L}{1-L}$$

➔ $$WACC = k_E(1-L) + k_D(1-t_C)L = k_A - k_A t_C L$$

Rule 2a: Debt rebalanced (Miles Ezzel)

- Assumption:
  - any cash flows
  - debt rebalanced $D_t/V_{L,t} = L$ (a constant)

$$V_{L,t} = \frac{FCF_{t+1} + V_{L,t+1}}{1+k_A} + \frac{k_D t_C L V_{L,t}}{1+k_D} \Rightarrow V_{L,t} = \frac{FCF_t + V_{L,t+1}}{1+k_A - k_D t_C L \frac{1+k_A}{1+k_D}}$$

$$V_{TS,t} = \left[\frac{k_D t_C L}{1+k_D(1-t_C)L}\right] V_{U,t} + \frac{V_{TS,t+1}}{1+k_A - k_D t_C L \frac{1+k_A}{1+k_D}}$$

→ $k_E = k_A + \left[k_A - k_D\left(1+t_C\left(\frac{k_A-k_D}{1+k_D}\right)\right)\right]\frac{L}{1-L}$

→ $WACC = k_E(1-L) + k_D(1-t_C)L = k_A - k_D t_C L \frac{1+k_A}{1+k_D}$

Miles-Ezzel: example

Data

Investment	300
Pre-tax CF	
Year 1	50
Year 2	100
Year 3	150
Year 4	100
Year 5	50
k_A	10%
k_D	5%
t_C	40%
L	25%

Base case NPV = -300 + 340.14 = +40.14

Using Miles-Ezzel formula

WACC = 10% - 0.25 x 0.40 x 5% x 1.10/1.05 = 9.48%

APV = -300 + 344.55 = 44.85

Initial debt: $D_0 = 0.25\ V_0 = (0.25)(344.55)=86.21$

Debt rebalanced each year:

Year	V_t	D_t
0	344.55	86.21
1	327.52	81.88
2	258.56	64.64
3	133.06	33.27
4	45.67	11.42

Using MM formula:

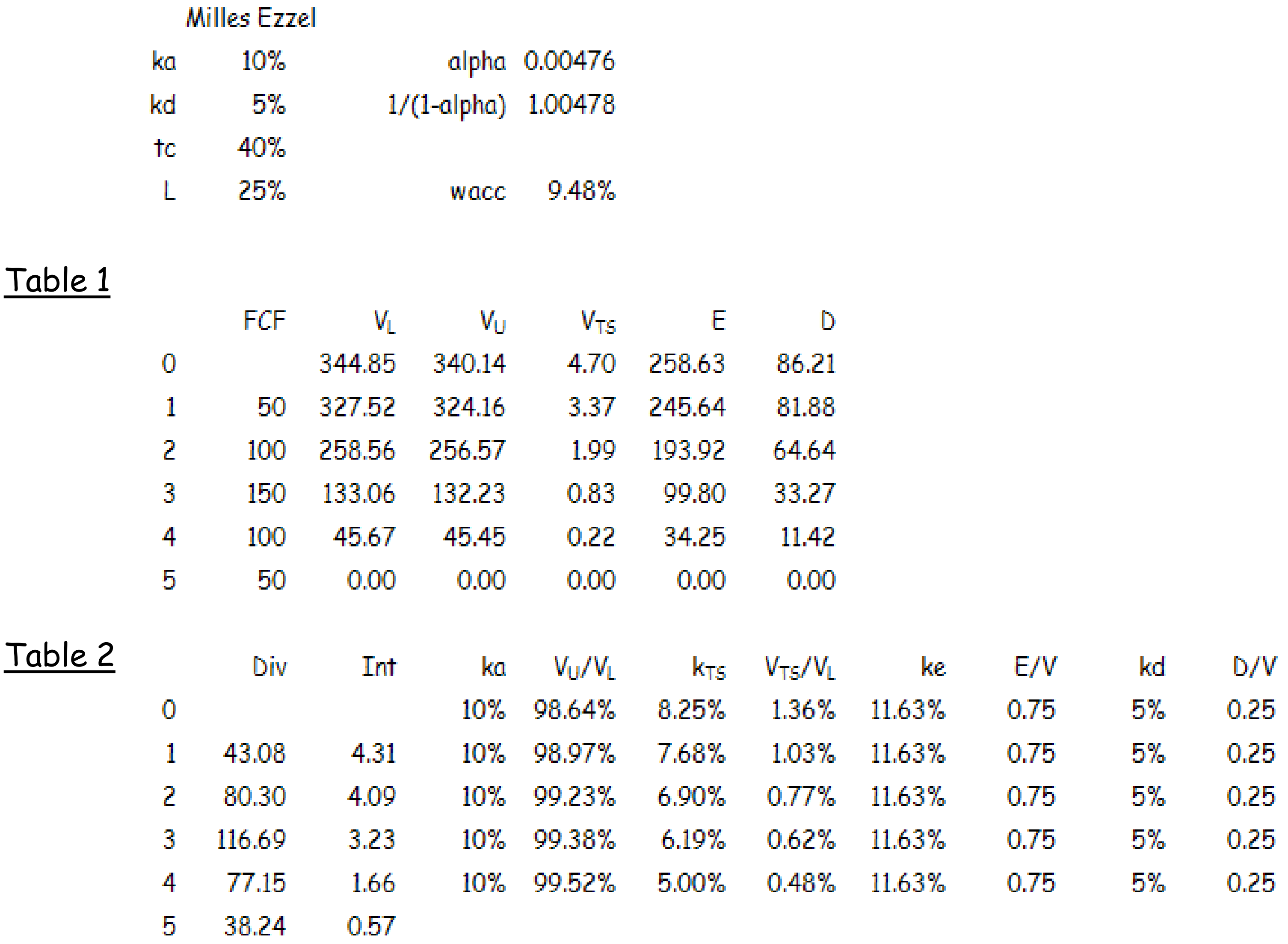
WACC = 10%(1-0.40 x 0.25) = 9%

APV = -300 + 349.21 = 49.21

Debt: $D = 0.25\ V = (0.25)(349.21) = 87.30$

No rebalancing

Miles-Ezzel: example



Milles Ezzel

ka	10%	alpha	0.00476
kd	5%	1/(1-alpha)	1.00478
tc	40%		
L	25%	wacc	9.48%

Table 1

	FCF	V_L	V_U	V_{TS}	E	D
0		344.85	340.14	4.70	258.63	86.21
1	50	327.52	324.16	3.37	245.64	81.88
2	100	258.56	256.57	1.99	193.92	64.64
3	150	133.06	132.23	0.83	99.80	33.27
4	100	45.67	45.45	0.22	34.25	11.42
5	50	0.00	0.00	0.00	0.00	0.00

Table 2

	Div	Int	ka	V_U/V_L	k_{TS}	V_{TS}/V_L	ke	E/V	kd	D/V
0			10%	98.64%	8.25%	1.36%	11.63%	0.75	5%	0.25
1	43.08	4.31	10%	98.97%	7.68%	1.03%	11.63%	0.75	5%	0.25
2	80.30	4.09	10%	99.23%	6.90%	0.77%	11.63%	0.75	5%	0.25
3	116.69	3.23	10%	99.38%	6.19%	0.62%	11.63%	0.75	5%	0.25
4	77.15	1.66	10%	99.52%	5.00%	0.48%	11.63%	0.75	5%	0.25
5	38.24	0.57								

Rule 2b: Debt rebalanced (Harris & Pringle)

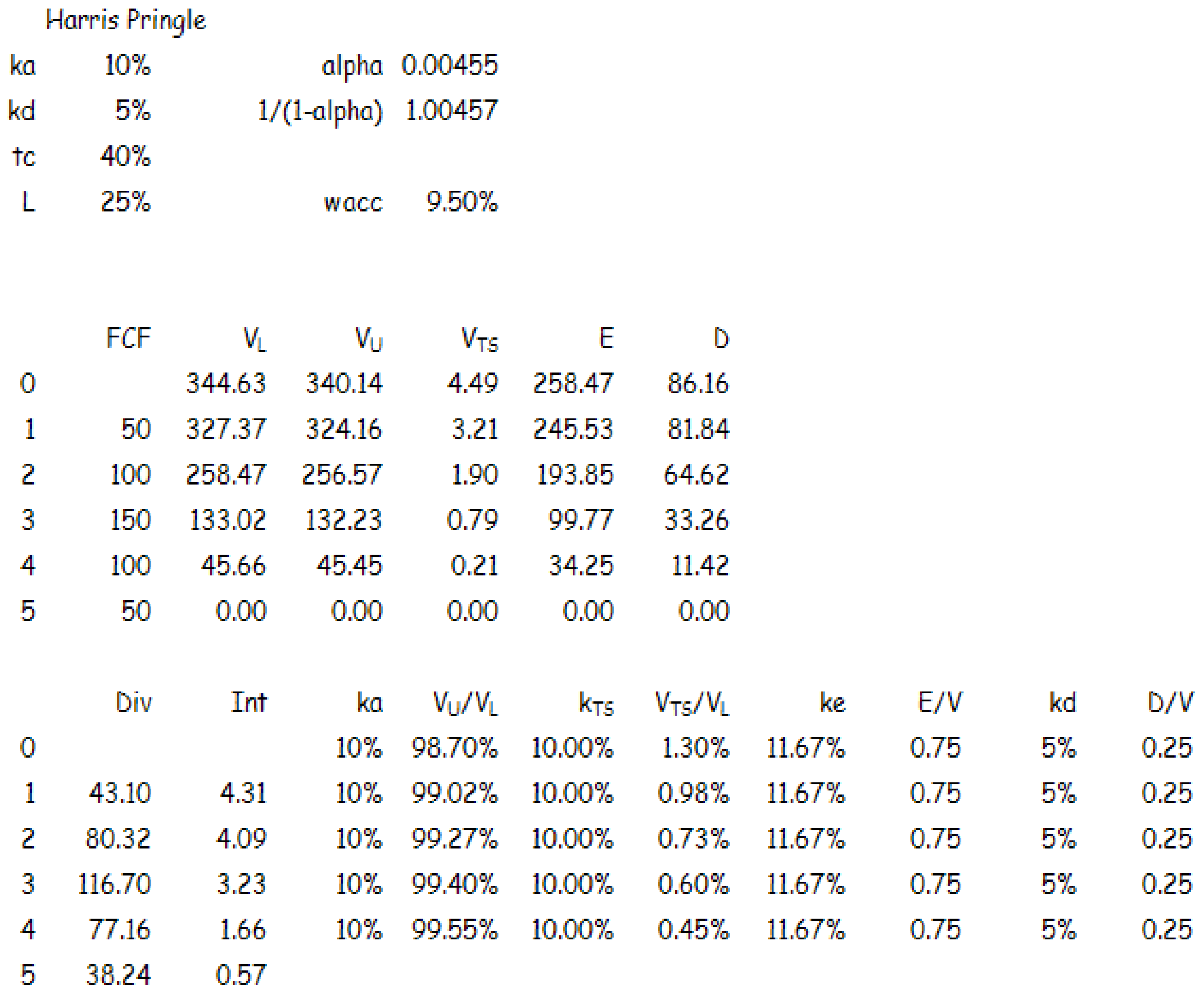
- Assumption:
  - » any free cash flows
  - » debt rebalanced continously $D_t = L\, V_{L,t}$
  - » the risk of the tax shield is equal to the risk of the unlevered firm
    - ✓ $k_{TS} = k_A$

$$V_{TS,t} = \left[\frac{k_D t_C L}{1+k_A(1-t_C)L}\right] V_{U,t} + \frac{V_{TS,t+1}}{1+k_A-k_D t_C L}$$

➔ $$k_E = k_A + (k_A - k_D)\frac{L}{1-L}$$

➔ $$WACC = k_E(1-L) + k_D(1-t_C)L = k_A - k_D t_C L$$

Harris-Pringle: example



Harris Pringle

ka	10%	alpha	0.00455
kd	5%	1/(1-alpha)	1.00457
tc	40%		
L	25%	wacc	9.50%

	FCF	V_L	V_U	V_{TS}	E	D
0		344.63	340.14	4.49	258.47	86.16
1	50	327.37	324.16	3.21	245.53	81.84
2	100	258.47	256.57	1.90	193.85	64.62
3	150	133.02	132.23	0.79	99.77	33.26
4	100	45.66	45.45	0.21	34.25	11.42
5	50	0.00	0.00	0.00	0.00	0.00

	Div	Int	ka	V_U/V_L	k_{TS}	V_{TS}/V_L	ke	E/V	kd	D/V
0			10%	98.70%	10.00%	1.30%	11.67%	0.75	5%	0.25
1	43.10	4.31	10%	99.02%	10.00%	0.98%	11.67%	0.75	5%	0.25
2	80.32	4.09	10%	99.27%	10.00%	0.73%	11.67%	0.75	5%	0.25
3	116.70	3.23	10%	99.40%	10.00%	0.60%	11.67%	0.75	5%	0.25
4	77.16	1.66	10%	99.55%	10.00%	0.45%	11.67%	0.75	5%	0.25
5	38.24	0.57								

Summary of Formulas

	Modigliani Miller	Miles Ezzel	Harris-Pringle
Operating CF	Perpetuity	Finite or Perpetual	Finite of Perpetual
Debt level	Certain	Uncertain	Uncertain
First tax shield	Certain	Certain	Uncertain
WACC $L = D/V$	$k_E(E/V) + k_D(1\text{-}t_C)(D/V)$		
	$k_A\,(1 - t_C\,L)$	$k_A - k_D t_C L \dfrac{1+k_A}{1+k_D}$	$k_A - k_D\,t_C\,L$
Cost of equity	$k_A+(k_A - k_D)(1\text{-}t_C)(D/E)$	$k_E = k_A + \left[k_A - k_D\left(1+t_C\left(\dfrac{k_A-k_D}{1+k_D}\right)\right)\right]\dfrac{L}{1-L}$	$k_A+(k_A - k_D)\ (D/E)$
Beta equity	$\beta_A+(\beta_A - \beta_D)\ (1\text{-}t_C)\ (D/E)$	$\beta_A\left(1+\dfrac{D}{E}\right)\left(\dfrac{1+k_D(1-t_C L)}{1+k_D}\right)$	$\beta_A + (\beta_A - \beta_D)\ (D/E)$

Source: Taggart – Consistent Valuation and Cost of Capital Expressions With Corporate and Personal Taxes *Financial Management* Autumn 1991

Constant perpetual growth

Which formula to use if unlevered free cash flows growth at a constant rate?

Growth	5%
Risk free rate	6%
Unlevered beta	1
Equity premium	4%
Beta debt	0.25
Tax rate	40%
Total asset	2,000
Initial debt	500
Initial free cash flow if g=0	192

$$V_0 = \frac{FCF_1}{WACC - g}$$

Unlevered cost of equity	10.0%
Cost of debt	7.00%
Initial free cash flow	92
Value of unlevered company	1,840

	MM	Miles-Ezzel	Harris-Pringle	Fernandez
L		23.50%	23.58%	
Value of tax shield	700	288	280	400
Value of levered company	2,540	2,128	2,120	2,240
Debt	500	500	500	500
Equity	2,040	1,628	1,620	1,740
WACC	8.62%	9.32%	9.34%	9.11%
Cost of equity	9.71%	10.90%	10.93%	10.52%
Cost of tax shield	7.00%	9.86%	10.00%	8.50%

Varying debt levels

- How to proceed if none of the financing rules applies?
- Two important instances:
    1. debt policy defined as an amount of borrowing instead of as a target percentage of value
    2. the amount of debt changes over time
- Use the Capital Cash Flow method suggested by Ruback
    - Ruback, Richard (1995), “A Note on Capital Cash Flow Valuation”, Harvard Business School, 9-295-069, January 1995.

Capital Cash Flow Valuation

- Assumptions:
  - CAPM holds
  - PV(Tax Shield) as risky as operating assets

$$V_{L,t-1} = \frac{FCF_t + V_{L,t}}{1 + WACC_t} \qquad WACC_t = k_A - k_D t_C \frac{D_{t-1}}{V_{L,t-1}}$$

$$V_{L,t-1}\left(1 + k_A - k_D t_C \frac{D_{t-1}}{V_{L,t-1}}\right) = FCF_t + V_{L,t}$$

$$V_{L,t-1} = \frac{FCF_t + k_D t_C D_{t-1} + V_{L,t}}{1 + k_A}$$

Capital cash flow
=FCF unlevered
+Tax shield

Capital Cash Flow Valuation: Example

ka	12%	Objective: L=	30%
Cost of debt	8%		
TaxRate	34%		

Income Statement	0	1	2	3	4	5
EBIT		20.00	25.00	30.00	30.00	30.00
Interest		6.40	5.86	5.32	4.79	4.25
Taxes		4.62	6.51	8.39	8.57	8.76
Net Income		8.98	12.63	16.29	16.64	17.00
Statement of CF						
OpCashFlow		8.98	12.63	16.29	16.64	17.00
Invest.Cash Flow		0	0	0	0	0
Dividend		2.25	5.91	9.56	9.92	17.00
Var Debt		-6.72	-6.72	-6.72	-6.72	0.00
Balance Sheet						
Assets	100.00	100.00	100.00	100.00	100.00	100.00
Debt	80	73.28	66.56	59.83	53.11	53.11
Equity	20	26.72	33.44	40.17	46.89	46.89
Vu	145.11	153.55	159.34	162.18	165.00	
WACC = ka-kd*tc*L					11.18%	
V					177.04	
D					53.11	
Capital Cash Flow		11.15	14.62	18.10	18.27	
V	158.62	166.50	171.85	174.38	177.04	

Bradley & Jarrell [2003] – constant growth model

- Bradley and Jarrell (BJ), “Inflation and the Constant-Growth Valuation Model: A Clarification”, Working Paper, February 2003
  - The most widely used valuation formula

$$V_0 = \frac{DIV_1}{k-g} = \frac{FCF_1}{k-g}$$

  - Solution of

$$V_0 = \frac{DIV_1}{1+r} + \frac{DIV_1(1+g)}{(1+r)^2} + \ldots + \frac{DIV_1(1+g)^{t-1}}{(1+r)^t} + \ldots$$

- Assumptions:
  - No inflation
  - All equity firm
- How to use this formula with inflation and debt?

Introducing inflation – no debt

- With no inflation, the real growth rate is

$$g = roi \times Plowback = roi \times (1 - Payout)$$

(*roi* is the real return on investment)

- With inflation, the nominal growth rate is:

$$G = ROI \times Plowback + (1 - Plowback) \times inflation$$

(*ROI* is the nominal return on investment)

Growth in nominal earnings - details

BJ(16) $$\Delta EBIAT_t = \Delta K_{t-1} \times roi \times (1+i)$$

BJ(17) $$K_t = (K_{t-1} - Dep_{t-1})(1+i) + CAPEX_t + WCR_t$$

$$REX_t = Dep_{t-1}(1+i)$$

$EBIAT = EBIT(1 - t_C)$
K = total capital (book value)
i = inflation rate
$CAPEX = REX + NNI$
REX = replacement expenditures
NNI = net new investments

BJ(20) $$\Delta EBIAT_t = i \times roi \times (1+i) \times K_{t-1} + (NNI_t + \Delta WCR_t) \times roi \times (1+i)$$

BJ(23) $$\boxed{G = i + Plowback \times roi \times (1+i)}$$

$$ROI = (1+roi)(1+i) - 1 = roi + i + roi \times i$$

BJ(27) $$\boxed{G = Plowback \times ROI + (1 - Plowback) \times i}$$

Valuing the company

$$V_0 = \frac{EBIAT_1(1 - Plowback)}{k_A - G} = \frac{ebiat_1(1 - Plowback)}{k_A - g}$$

Using
nominal
values

Using
real
values

Same result

Debt - which WACC formula to use?

- The Miles and Ezzell (M&E) holds in nominal terms.

$$V_0 = \frac{FCF_1}{WACC - G}$$

- With:

$$WACC = k_A - t_C k_D L \frac{1+k_A}{1+k_D}$$

- The value of a levered firm is positively related to the rate of inflation

Interest tax shield and inflation

Borrow	€1,000 for 1 year
Real cost of debt	3%
Tax rate	40%
1. Inflation	**0%**
Interest year 1	€30
Tax shield	€12
2. Suppose inflation =	**2%**
Nominal cost of debt	5.06%
Nominal interest year 1	€50.60
Nominal tax shield	€20.24
Real tax shield	€19.84

	Borrow	Repay
Nominal	€1,000.0	€1,000.0
Real	€1,000.0	€980.4
Difference		-€19.6

This difference is compensated by a higher interest

Nominal interest year 1	€50.6
Real interest (adjusted for inflation)	€30.60
Repayment of real principal	€20.00

Repayment of real principal is tax deductible →higher tax shield